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# REVIEW

**Mathematical Theory of Transport Processes in Gases.** BY J. H. FERZIGER and H. G. KAPER. North-Holland, 1972. 579 pp. H£1.120.

In the hundred years since Boltzmann obtained his famous nonlinear equation for the rate of change of the velocity distribution function $F_1$ in a dilute gas, the theory of the transport of mechanical shear forces, heat, electricity and other continuum properties through gases has been developed in fits and starts by a large number of scientists and applied mathematicians. The principal difficulty is that, save for some rather special molecular models, it is necessary to solve Boltzmann's equation to calculate transport coefficients like viscosity and conductivity, and, apart from Maxwell's equilibrium solution $F_1^0$, no exact solution exists. Just over 50 years ago Enskog and Chapman independently devised series solutions of the equation, but the algebra is so complicated, although not difficult, that texts are usually content to obtain the first term beyond $F_1^0$. This expansion remains the corner-stone of the mathematical theory of transport processes, although the advent of high-speed computers has permitted other developments. The subject has attracted great interest over the last 25 years because of its relevance to high-altitude flight, shock-wave structure, the behaviour of ionized gases and other situations in which a purely fluid description may fail.

As described in chapter 1 (mainly a historical survey), the book under review is a detailed account of the use of Boltzmann's equation and its generalizations to derive transport properties of gases. Chapter 2 on the "properties of a gas' introduces the usual definitions of density, pressure and internal energy in terms of the one-particle distribution function $F_1$. As with most authors, they regrettably lump together the streaming energy of diffusing species with the genuine internal energy of the species' random motion. Entropy, which plays a role in later chapters, could have been usefully introduced at this stage. Chapter 3 on "Boltzmann's equation" commences with the classical derivation of the equation via the molecular-chaos hypothesis and contains the now common sleight-of-hand trick (p. 30) in which the element $d\epsilon$ of impact azimuthal angle for a direct collision is falsely equated to its value $d\epsilon'$ for an inverse collision (see E. Dahlberg in *J. Phys.* A, shortly). There follows a clear account of the BBGKY-hieracy of equations with the object of showing how the irreversible Boltzmann equation may be derived from the reversible Liouville equation. Some *physical* hypothesis corresponding to molecular chaos is necessary in this derivation and in the text is taken to be a neglect of particle correlations at some initial instant of time. Other *ansätze* could have been used – this reviewer finds the assumption of disparate time scales as in Bogoliubov's approach more compelling – but there is no doubt of the authors' command of this difficult topic.

Chapter 4 is on "Fundamental properties of Boltzmann's equation" and offers the usual treatment of the $H$-theorem. The authors ignore the concept of kinetic entropy, which holds for non-equilibrium distributions, and are a little too ambitious, throwing in incomplete snippets here and there, e.g. their remarks on information theory and statistical mechanics. There are easier, more direct approaches to the $H$-theorem in bounded systems (via the concept of irreversible entropy production per unit volume) than the rather heavy account of §4.5. The chapter finishes with an existence theorem for the linear Boltzmann equation; the reader will require some functional analysis and at a critical point another text to complete the theorem itself. Chapters 5 and 6 deal with the non-uniform states of a simple gas and of a gas mixture respectively, providing good accounts of the Chapman–Enskog expansion, Hilbert's theory and the contrast between these methods. Burnett's second-order theory is outlined and chapter 6 concludes with an account of the Lorentz gas.

The next three chapters deal with the details of the application of the Chapman–Enskog expansion to the calculation of transport coefficients in terms of prescribed intermolecular potentials, several model potentials being discussed. Chapter 10 on comparison of theory and experiment is particularly welcome and interesting. It contains many experimental results for the noble gases, these being the only ones to which the theory of the earlier chapters can be applied without further approximation.

Chapter 11 deals with the complications of polyatomic gases, which have internal degrees of freedom and asymmetric molecular interactions. The chapter starts by discussing the effect of asymmetric potentials on viscosity and follows with the Eucken correction, which allows for the fact that thermal conductivity is contributed to differently by the molecular internal energy and the molecular translatory energy. In §11.4 there is a good survey of quantum-mechanical effects which *inter alia* produce anisotropic transport coefficients. Likewise as described at the end of the chapter, magnetic fields also produce such coefficients. It seems a pity that both the general phenomenological theory of transport coefficients in an anisotropic medium and the Onsager–Casimir reciprocal relations were not treated earlier in the text, as these theories are more general than the particular model used here.

The next two chapters are concerned with dense gases. The first gives Enskog's corrections to dilute gas theory and shows by comparison with experiment that the modified transport coefficients are accurate for at least moderately dense gases. The second contains an account of recent work on the problem of generalizing Boltzmann's collision integral to include dense gas effects. Bogoliubov's functional assumption, namely that the time dependence of the two-particle distribution function is adequately represented via the one-particle distribution, is adopted and a generalized Boltzmann equation derived, which is then solved by the usual Chapman–Enskog method. A full account of the hypothesis involved in the theory is given, with many references to recent studies.

Chapter 14 deals with ionized gases and is more of a survey than a detailed account of the Chapman–Enskog method of dealing with such gases. The difficulties arising from the long-ranging Coulomb potential are clearly explained, as are also the additional complications – especially the anisotropy produced by a magnetic field. The final chapter is on the dynamics of rarefied gases. It contains a good account of specular and diffuse reflexion. The slip flow problem is treated by the complex variable approach used by Van Kampen and Case for plasma waves. There are several appendices including extensive tables of various transport integrals.

Despite the few criticisms this reviewer has made above, the book is really an outstanding contribution by authors who are well experienced in their subject. It is certainly recommended to workers in the kinetic theory of gases who want a comprehensive account of transport theory with references to recent researchers.

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